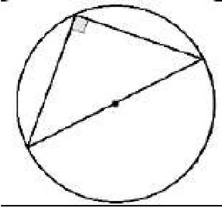
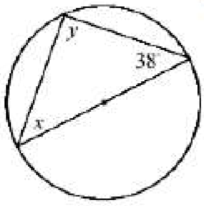
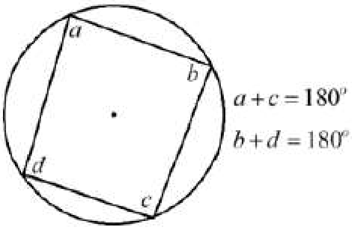
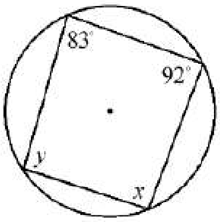
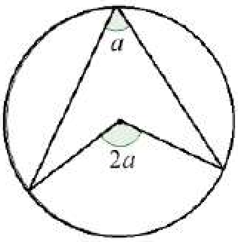
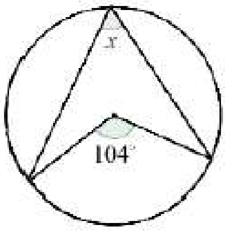
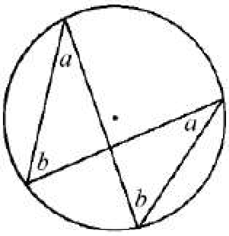
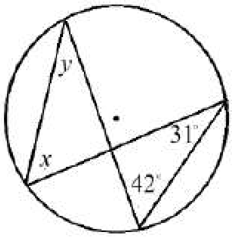
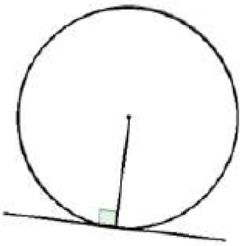
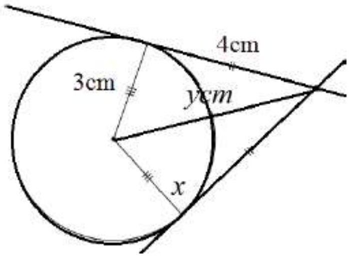
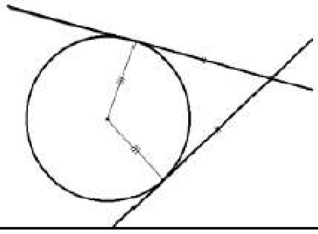
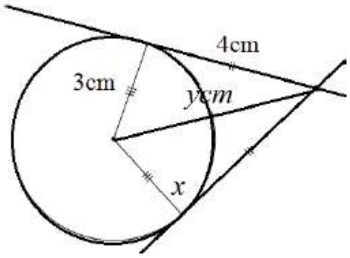
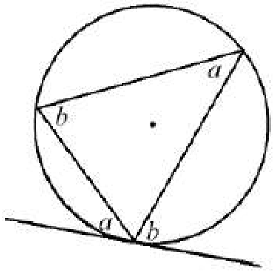
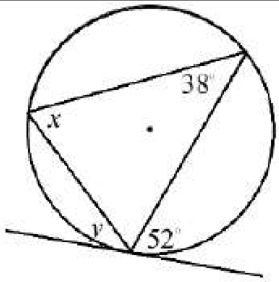
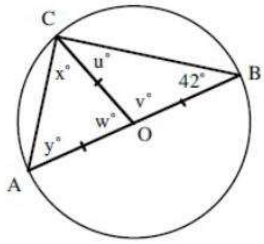
Knowledge Organiser Y11 H Maths Circle Theorems

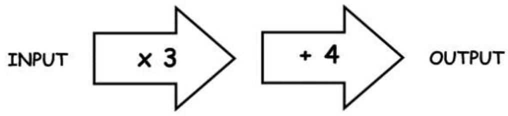
Key Vocabulary	Definition/Tips	Example
Semi-circle	A half of a circle or of its circumference.	
Quadrilateral	A four-sided shape	
Circle Theorem 1	Angles in a semi-circle have a right angle at the circumference. 	 $y = 90°$ $x = 180 - 90 - 38 = 52°$
Circle Theorem 2	Opposite angles in a cyclic quadrilateral add up to 180°. 	 $x = 180 - 83 = 97°$ $y = 180 - 92 = 88°$
Circle Theorem 3	The angle at the centre is twice the angle at the circumference. 	 $x = 104 \div 2 = 52°$
Circle Theorem 4	Angles in the same segment are equal. 	 $x = 42°$ $y = 31°$
Circle Theorem 5	A tangent is perpendicular to the radius at the point of contact. 	 $y = 5cm$ (Pythagoras' Theorem)

Circle Theorem 6	Tangents from an external point at equal in length. 	 $x = 90°$
Circle Theorem 7	Alternate Segment Theorem 	 $x = 52°$ $y = 38°$
Circle Theorem 7	Base angles of isosceles triangles are equal 	

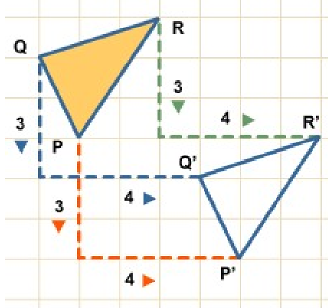
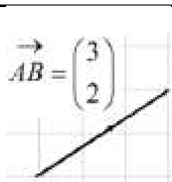
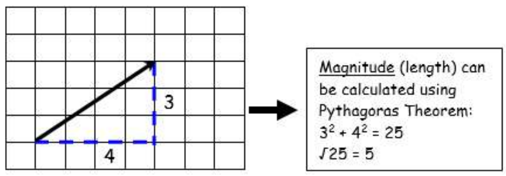
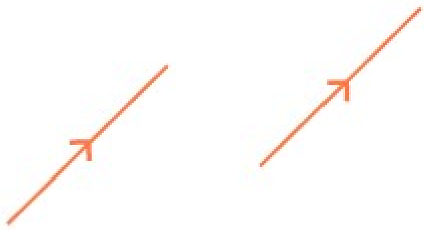
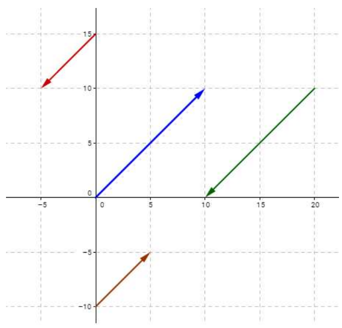
Knowledge Organiser Y11 Maths Formulae, Algebraic Fraction and Surd

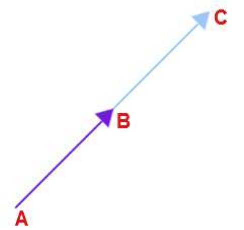
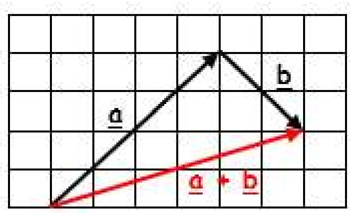
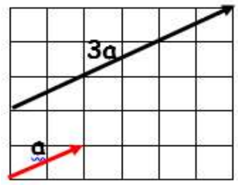
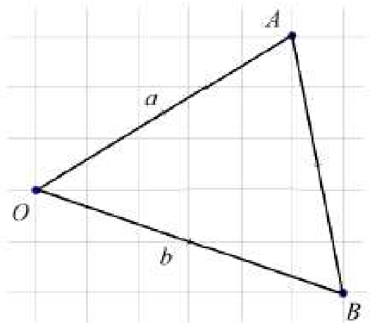
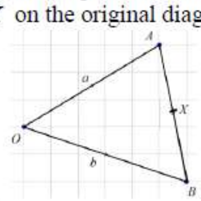
Key Vocabulary	Definition/Tips	Example
Solve	To find the **answer**/value of something **Use inverse operations** on both sides of the equation (balancing method) until you find the value for the letter.	Solve $2x - 3 = 7$ Add 3 on both sides $2x = 10$ Divide by 2 on both sides $x = 5$
Inverse	**Opposite**	The inverse of addition is subtraction. The inverse of multiplication is division.
Rearranging Formulae	**Use inverse operations** on both sides of the formula (balancing method) until you find the expression for the letter.	Make x the subject of $y = \frac{2x-1}{z}$ Multiply both sides by z $yz = 2x - 1$ Add 1 to both sides $yz + 1 = 2x$ Divide by 2 on both sides $\frac{yz+1}{2} = x$ We now have x as the subject.
Writing Formulae	**Substitute letters for words** in the question.	Bob charges £3 per window and a £5 call out charge. $C = 3N + 5$ Where N=number of windows and C=cost
Substitution	**Replace letters with numbers**. Be careful of $5x^2$. You need to square first, then multiply by 5.	$a = 3, b = 2$ and $c = 5$. Find: 1. $2a = 2 \times 3 = 6$ 2. $3a - 2b = 3 \times 3 - 2 \times 2 = 5$ 3. $7b^2 - 5 = 7 \times 2^2 - 5 = 23$
Algebraic Fraction	A fraction whose **numerator** and **denominator** are **algebraic expressions.**	$\frac{6x}{3x-1}$
Adding/ Subtracting Algebraic Fractions	For $\frac{a}{b} \pm \frac{c}{d}$, the **common denominator** is bd $\frac{a}{b} \pm \frac{c}{d} = \frac{ad}{bd} \pm \frac{bc}{bd} = \frac{ad \pm bc}{bd}$	$\frac{1}{x} + \frac{x}{2y}$ $= \frac{1(2y)}{2xy} + \frac{x(x)}{2xy}$ $= \frac{2y + x^2}{2xy}$
Multiplying Algebraic Fractions	**Multiply** the **numerators together** and the **denominators together.** $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$	$\frac{x}{3} \times \frac{x+2}{x-2}$ $= \frac{x(x+2)}{3(x-2)}$

		$= \frac{x^2 + 2x}{3x - 6}$
Dividing Algebraic Fractions	**Multiply** the first fraction by the **reciprocal of the second fraction.** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$	$\frac{x}{3} \div \frac{2x}{7}$ $= \frac{x}{3} \times \frac{7}{2x}$ $= \frac{7x}{6x} = \frac{7}{6}$
Simplifying Algebraic Fractions	**Factorise** the numerator and denominator and **cancel common factors.**	$\frac{x^2 + x - 6}{2x - 4} = \frac{(x+3)(x-2)}{2(x-2)} = \frac{x+3}{2}$
Rational Number	A number of the form $\frac{p}{q}$, where p **and** q **are integers** and $q \neq 0$. A number that cannot be written in this form is called an 'irrational' number	$\frac{4}{9}, 6, -\frac{1}{3}, \sqrt{25}$ are examples of rational numbers. $\pi, \sqrt{2}$ are examples of an irrational numbers.
Surd	The **irrational number** that is a **root of a positive integer,** whose value cannot be determined exactly. Surds have **infinite non-recurring decimals.**	$\sqrt{2}$ is a surd because it is a root which cannot be determined exactly. $\sqrt{2} = 1.41421356\ldots$ which never repeats.
Rules of Surds	$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ $a\sqrt{c} \pm b\sqrt{c} = (a \pm b)\sqrt{c}$ $\sqrt{a} \times \sqrt{a} = a$	$\sqrt{48} = \sqrt{16} \times \sqrt{3} = 4\sqrt{3}$ $\sqrt{\frac{25}{36}} = \frac{\sqrt{25}}{\sqrt{36}} = \frac{5}{6}$ $2\sqrt{5} + 7\sqrt{5} = 9\sqrt{5}$ $\sqrt{7} \times \sqrt{7} = 7$
Rationalise a Denominator	The process of rewriting a fraction so that the **denominator contains only rational numbers.**	$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{6}}{2}$ $\frac{6}{3+\sqrt{7}} = \frac{6(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})}$ $= \frac{18 - 6\sqrt{7}}{9 - 7}$ $= \frac{18 - 6\sqrt{7}}{2} = 9 - 3\sqrt{7}$

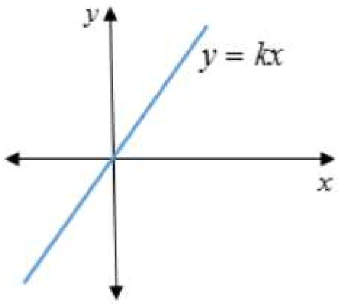
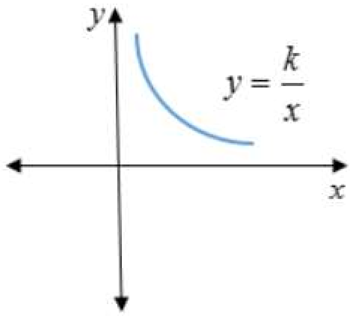
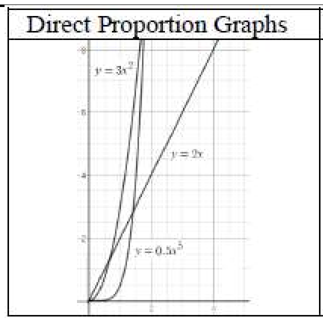
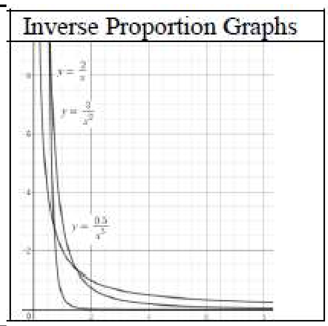
Key vocabulary	Definition/Tips	Example
Function Machine	Takes an **input** value, performs some **operations** and produces an **output** value.	
Function	A **relationship** between two sets of values.	$f(x) = 3x^2 - 5$ 'For any input value, square the term, then multiply by 3, then subtract 5'.
Function notation	$f(x)$ $\boldsymbol{x}$ is the **input** value $\boldsymbol{f(x)}$ is the **output** value.	$f(x) = 3x + 11$ Suppose the input value is $x = 5$ The output value is $f(5) = 3 \times 5 + 11 = 26$
Inverse function	$f^{-1}(x)$ A function that performs the **opposite process** of the original function. 1. Write the function as $y = f(x)$ 2. Rearrange to make x the subject. 3. Replace the $\boldsymbol{y}$ **with** $\boldsymbol{x}$ and the $\boldsymbol{x}$ **with** $\boldsymbol{f^{-1}(x)}$	$f(x) = (1 - 2x)^5$. Find the inverse. $y = (1 - 2x)^5$ $\sqrt[5]{y} = 1 - 2x$ $1 - \sqrt[5]{y} = 2x$ $\frac{1 - \sqrt[5]{y}}{2} = x$ $f^{-1}(x) = \frac{1 - \sqrt[5]{x}}{2}$
Composite function	A **combination** of two or more **functions** to create a new function. $\boldsymbol{fg(x)}$ is the composite function that **substitutes** the function $\boldsymbol{g(x)}$ **into** the function $\boldsymbol{f(x)}$. $\boldsymbol{fg(x)}$ means '**do g first, then f**' $\boldsymbol{gf(x)}$ means '**do f first, then g**'	$f(x) = 5x - 3,\ g(x) = \frac{1}{2}x + 1$ What is $fg(4)$? $g(4) = \frac{1}{2} \times 4 + 1 = 3$ $f(3) = 5 \times 3 - 3 = 12 = fg(4)$ What is $fg(x)$? $fg(x) = 5\left(\frac{1}{2}x + 1\right) - 3 = \frac{5}{2}x + 2$
Expression	A mathematical statement written using **symbols**, **numbers** or **letters**,	$3x + 2$ or $5y^2$
Equation	A statement showing that **two expressions are equal**	2y – 17 = 15
Identity	An equation that is **true for all values** of the variables An identity uses the symbol: ≡	*2x ≡ x+x*
Formula	Shows the **relationship** between **two or more variables**	Area of a rectangle = length x width or A= LxW
Coefficient	A **number** used to **multiply** a **variable**.	6z

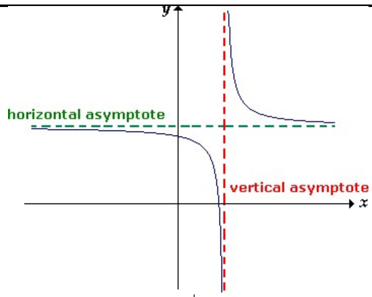
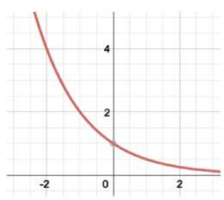
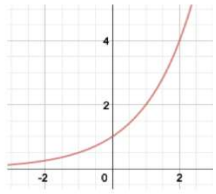
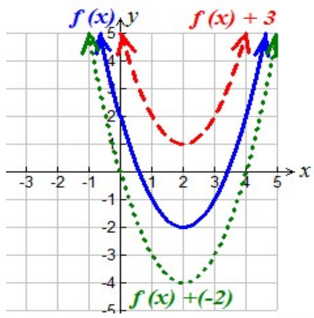
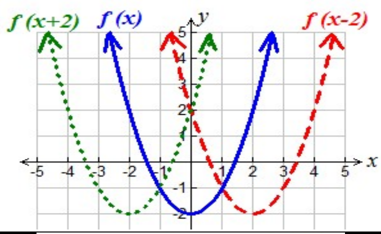
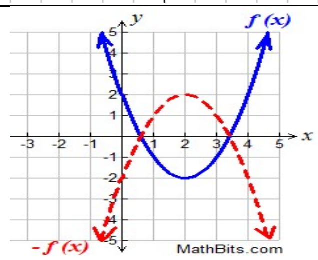
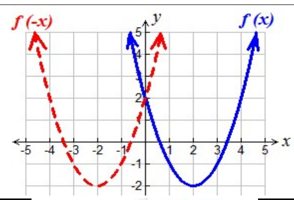
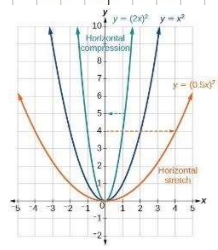
	It is the number that comes before/in front of a letter.	6 is the coefficient z is the variable
Odds and Evens	An **even** number is a **multiple of 2** An **odd** number is an integer which is **not a multiple of 2**.	If n is an integer (whole number): An even number can be represented by **2n** or **2m** etc. An odd number can be represented by **2n-1** or **2n+1** or **2m+1** etc.
Consecutive Integers	Whole numbers that follow each other in order.	If n is an integer: **n, n+1, n+2** etc. are consecutive integers.
Square Terms	A term that is produced by multiply another term by itself.	If n is an integer: n^2, m^2 etc. are square integers
Sum	The sum of two or more numbers is the value you get when you add them together.	The sum of 4 and 6 is 10
Product	The product of two or more numbers is the value you get when you multiply them together.	The product of 4 and 6 is 24
Multiple	To show that an expression is a **multiple** of a number, you need to show that you can **factor out the number**.	$4n^2 + 8n - 12$ is a multiple of 4 because it can be written as: $4(n^2 + 2n - 3)$

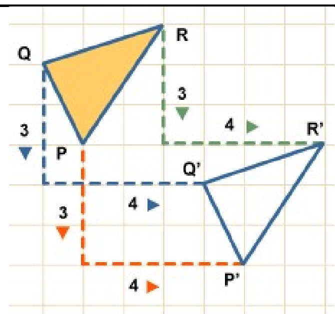
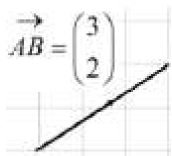
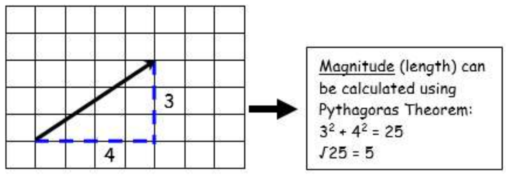
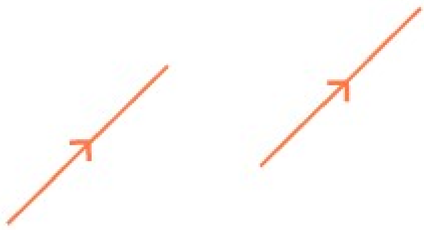
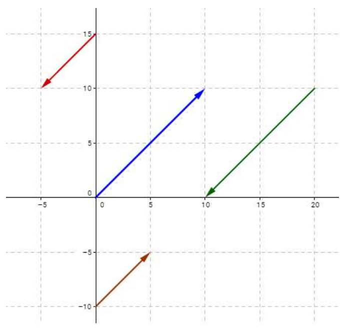
Key vocabulary	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape.** The shape does not change **size** or **orientation.**	
2. Vector Notation	A vector can be written in 3 ways: **a** or $\overrightarrow{\boldsymbol{AB}}$ or $\begin{pmatrix}\mathbf{1}\\\mathbf{3}\end{pmatrix}$	
3. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix}2\\3\end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix}-1\\-5\end{pmatrix}$ means '1 left, 5 down'
4. Vector	A **vector** is a quantity represented by an arrow with both **direction** and **magnitude.** $\overrightarrow{AB} = -\overrightarrow{BA}$	
5. Magnitude	Magnitude is defined as the **length** of a vector.	
6. Equal Vectors	If two vectors have the **same magnitude and direction**, they are **equal**.	
7. Parallel Vectors	**Parallel** vectors are **multiples** of each other.	2**a**+**b** and 4**a**+2**b** are parallel as they are multiple of each other. 

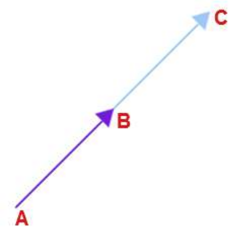
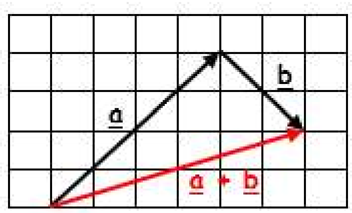
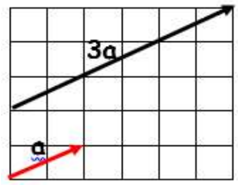
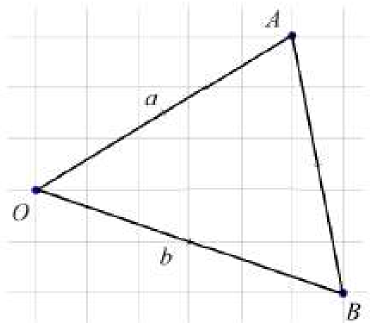
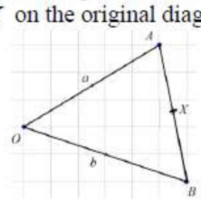
8. Collinear Vectors	**Collinear** vectors are vectors that are on the **same line.** To show that two vectors are **collinear**, show that one vector is a **multiple** of the other (parallel) **AND** that both vectors **share a point.**	
9. Resultant Vector	The **resultant** vector is the vector that results from **adding** two or more vectors together. The resultant can also be shown by **lining up** the **head** of one vector with the **tail** of the other.	if **a** = $\binom{4}{4}$ and **b** = $\binom{2}{-2}$ then **a** + **b** = $\binom{4}{4}$ + $\binom{2}{-2}$ = $\binom{6}{2}$ 
10. Scalar of a Vector	A **scalar** is the **number** we **multiply** a vector by.	 Example: 3a + 2b = = $3\binom{2}{1} + 2\binom{4}{-1}$ = $\binom{6}{3} + \binom{8}{-2}$ = $\binom{14}{1}$
11. Vector Geometry	 $\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$ $\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$ $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$	**Example 1:** X is the midpoint of AB. Find $\overrightarrow{OX}$ **Answer:** Draw X on the original diagram  Now build up a journey. You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$. This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b-a)$. This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a+b)$

Knowledge Organisers: Proportion Graph Transformations

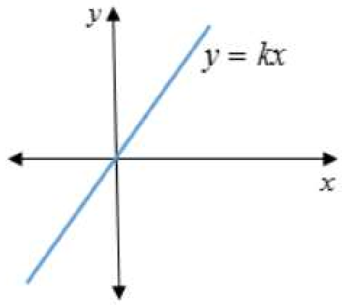
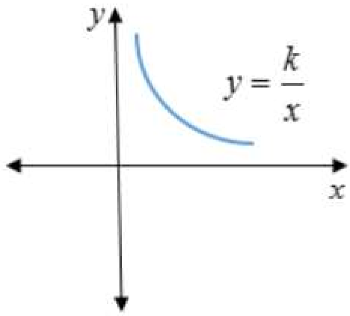
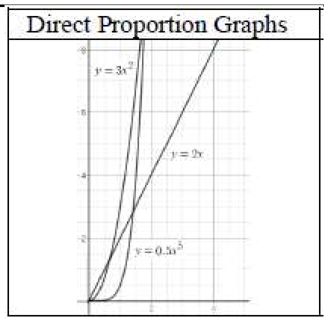
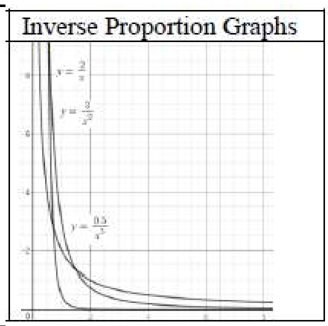
Key Vocabulary	Definition/Tips	Example
Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality.**	
Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
Using proportionality formulae	**Direct**: **y = kx** or **y** $\propto$ **x** **Inverse**: **y =** $\boldsymbol{\frac{k}{x}}$ or **y** $\propto \boldsymbol{\frac{1}{x}}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value.**	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60
Direct Proportion with powers	Graphs showing **direct proportion** can be written in the form $\boldsymbol{y = kx^n}$ Direct proportion graphs will always start at the origin.	
Inverse Proportion with powers	Graphs showing **inverse proportion** can be written in the form $\boldsymbol{y = \frac{k}{x^n}}$ Inverse proportion graphs will never start at the origin.	

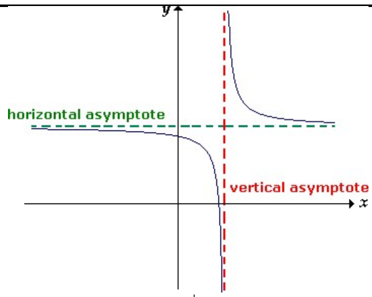
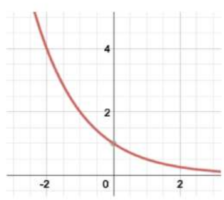
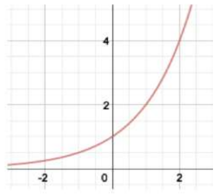
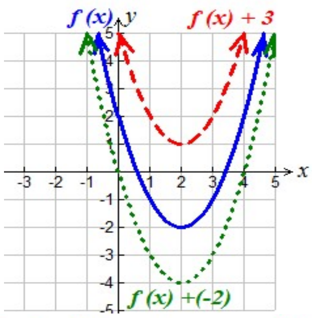
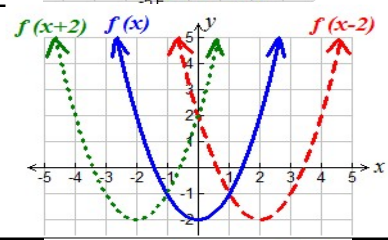
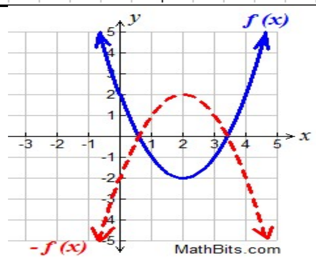
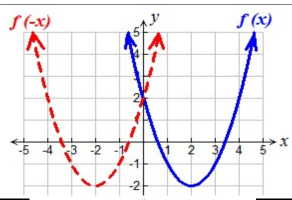
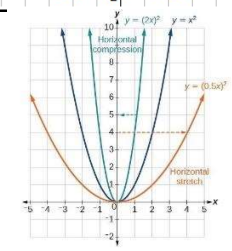
Asymptote	A **straight line** that a graph **approaches** but **never touches**.	
Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$, where a is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases.** The graph has an **asymptote** which is the **x-axis.**	
$f(x) + a$	**Vertical translation** up a units. $\begin{pmatrix}0\\a\end{pmatrix}$	
$f(x + a)$	**Horizontal translation** <u>left</u> a units. $\begin{pmatrix}-a\\0\end{pmatrix}$	
. $-f(x$	**Reflection** over the **x-axis.**	
$f(-x)$	**Reflection** over the **y-axis.**	
$af(x)$	**Vertical stretch for │a│> 1** **Vertical compression for 0<│a│< 1**	
$f(bx)$	**Horizontal compression for │b│> 1** **Horizontal stretch for 0<│b│< 1**	

Key vocabulary	Definition/Tips	Example
1. Translation	**Translate** means to **move a shape.** The shape does not change **size** or **orientation.**	
2. Vector Notation	A vector can be written in 3 ways: **a** or $\overrightarrow{\boldsymbol{AB}}$ or $\begin{pmatrix} \mathbf{1} \\ \mathbf{3} \end{pmatrix}$	
3. Column Vector	In a column vector, the **top** number moves **left (-) or right (+)** and the **bottom** number moves **up (+) or down (-)**	$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means '2 right, 3 up' $\begin{pmatrix} -1 \\ -5 \end{pmatrix}$ means '1 left, 5 down'
4. Vector	A **vector** is a quantity represented by an arrow with both **direction** and **magnitude.** $\overrightarrow{AB} = -\overrightarrow{BA}$	
5. Magnitude	Magnitude is defined as the **length** of a vector.	
6. Equal Vectors	If two vectors have the **same magnitude and direction**, they are **equal.**	
7. Parallel Vectors	**Parallel** vectors are **multiples** of each other.	2**a**+**b** and 4**a**+2**b** are parallel as they are multiple of each other. 

8. Collinear Vectors	**Collinear** vectors are vectors that are on the **same line.** To show that two vectors are **collinear**, show that one vector is a **multiple** of the other (parallel) **AND** that both vectors **share a point.**	
9. Resultant Vector	The **resultant** vector is the vector that results from **adding** two or more vectors together. The resultant can also be shown by **lining up** the **head** of one vector with the **tail** of the other.	if **a** = $\binom{4}{4}$ and **b** = $\binom{2}{-2}$ then **a** + **b** = $\binom{4}{4} + \binom{2}{-2} = \binom{6}{2}$ 
10. Scalar of a Vector	A **scalar** is the **number** we **multiply** a vector by.	 Example: 3a + 2b = $= 3\binom{2}{1} + 2\binom{4}{-1}$ $= \binom{6}{3} + \binom{8}{-2}$ $= \binom{14}{1}$
11. Vector Geometry	 $\overrightarrow{OA} = a \quad \overrightarrow{AO} = -a$ $\overrightarrow{OB} = b \quad \overrightarrow{BO} = -b$ $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -a + b = b - a$ $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -b + a = a - b$	**Example 1:** X is the midpoint of AB. Find $\overrightarrow{OX}$ **Answer:** Draw X on the original diagram  Now build up a journey. You could use $\overrightarrow{OX} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$. This will give: $\overrightarrow{OX} = a + \frac{1}{2}(b-a)$. This will simplify to $\frac{1}{2}a + \frac{1}{2}b$ or $\frac{1}{2}(a+b)$

Knowledge Organisers: Proportion Graph Transformations

Key Vocabulary	Definition/Tips	Example
Direct Proportion	If two quantities are in direct proportion, **as one increases**, the **other increases** by the **same percentage**. If y is directly proportional to x, this can be written as $\boldsymbol{y \propto x}$ An equation of the form $\boldsymbol{y = kx}$ represents direct proportion, where k **is the constant of proportionality.**	
Inverse Proportion	If two quantities are inversely proportional, **as one increases**, the **other decreases** by the **same percentage**. If y is inversely proportional to x, this can be written as $\boldsymbol{y \propto \frac{1}{x}}$ An equation of the form $\boldsymbol{y = \frac{k}{x}}$ represents inverse proportion.	
Using proportionality formulae	**Direct**: **y = kx** or **y** ∝ **x** **Inverse**: **y =** $\boldsymbol{\frac{k}{x}}$ or **y** ∝ $\boldsymbol{\frac{1}{x}}$ 1. **Solve to find k** using the pair of values in the question. 2. **Rewrite the equation** using the k you have just found. 3. **Substitute the other given value** from the question in to the equation to **find the missing value.**	p is directly proportional to q. When p = 12, q = 4. Find p when q = 20. 1. p = kq 12 = k x 4 so k = 3 2. p = 3q 3. p = 3 x 20 = 60, so p = 60
Direct Proportion with powers	Graphs showing **direct proportion** can be written in the form $\boldsymbol{y = kx^n}$ Direct proportion graphs will always start at the origin.	
Inverse Proportion with powers	Graphs showing **inverse proportion** can be written in the form $\boldsymbol{y = \frac{k}{x^n}}$ Inverse proportion graphs will never start at the origin.	

Asymptote	A **straight line** that a graph **approaches** but **never touches**.	
Exponential Graph	The equation is of the form $\boldsymbol{y = a^x}$, where a is a number called the **base**. If $\boldsymbol{a > 1}$ the graph **increases**. If $\boldsymbol{0 < a < 1}$, the graph **decreases.** The graph has an **asymptote** which is the **x-axis.**	 
$f(x) + a$	**Vertical translation** up a units. $\begin{pmatrix} 0 \\ a \end{pmatrix}$	
$f(x + a)$	**Horizontal translation** <u>left</u> a units. $\begin{pmatrix} -a \\ 0 \end{pmatrix}$	
. $-f(x$	**Reflection** over the **x-axis.**	
$f(-x)$	**Reflection** over the **y-axis.**	
$af(x)$	**Vertical stretch for │a│> 1** **Vertical compression for 0<│a│< 1**	
$f(bx)$	**Horizontal compression for │b│> 1** **Horizontal stretch for 0<│b│< 1**	