GCSE Mathematics

*Threshold concepts*

*for linear 5 yr Maths Course (Grades 1-9)*

*Version 14/05/17*

*Mapping 6 Pathways*

|  |  |
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| *All (G6), Most (G7), Some (G8-9)* | **Most Able (G&T)** |
| Y7 | Y8 | Y9 | Y10-11 |
| *All (G5), Most (G6), Some (G6-7)* | **HIGHER** |  |
| Y7 | Y8 | Y9 | Y10-11 |
| *All (G3), Most (G4), Some (G5-6)* | **MIDDLE (H)** |  |
| Y7 | Y8 | Y9 | Y10-11 |
| *All (G2), Most (G3), Some (G4)* | **LOWER** |  |
| Y7 | Y8 | Y9 | Y10-11 |
|  |  STAGE 0 | STAGE 1 | STAGE 2 | STAGE 3 | STAGE 4 | STAGE 5 | STAGE 6 | STAGE 7 | STAGE 8 |
|  | G to F | E | D | C- | C+ | B+ | A | A\* |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 and 9 |

GCSE NUMBER

*Threshold concepts for 5 yr Maths Course (Grades 1-9)*

*Version 29/06/15*

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| **NUMBER 1** |
| **1** | **2** | **3** | **4** |
| Students should be able to demonstrate fluency in functional mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Limited Confidence*** | **Fluency - Increasing ability to use and apply standard techniques** Grade 2 Students should, for Grade 1 and functional mathematical topics, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts**Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Numbers, Powers and Decimals*** Read, write and order integers up to 4 digits
* Order, add and subtract positive and negative numbers (integers) in context.
* Partition and round whole numbers up to the nearest 1000
* use the symbols <, > and understand the ≠ symbol
* recall multiplications facts up to 10 x 10 and derive the corresponding division facts
* use mental methods to: find remainders after division
* use efficient written methods to add and subtract whole numbers
* use efficient written methods to multiply and divide 2-digit or 3-digit numbers by a single-digit number
* multiply and divide whole numbers by powers of ten
* begin to use the order of operations
* understand and use inverse operations
* Compare and order decimals with up to two places using the symbols < and >
* I can write decimals using tenths and hundredths
* Partition and identify the value of digits up to 1000
* identify squares of numbers to 10 × 10
* recognise odd and even numbers
* list the first 10 prime numbers
* recognise multiples and use simple tests of divisibility
* recognise multiples and factors, and identify factors of 2-digit numbers.
 | **Numbers, Powers and Decimals*** understand negative numbers as positions on a number line
* order, add and subtract positive and negative integers in context
* use the symbols <, > and understand the ≠ symbol in the context of integers and decimals
* round whole numbers to the nearest 10, 100 or 1000 and decimals to the nearest whole number or one decimal place
* multiply and divide 3-digit by 2-digit whole numbers
* use the order of operations, including brackets.
* use inverse operations in the context of integers
* consolidate recall of multiplication to 10 × 10 and quickly derive division facts
* multiply any integer or decimal by powers of ten
* understand and use decimal notation and place value
* compare and order decimals in different contexts
* use efficient written methods to: add and subtract whole numbers
* recognise square numbers to at least 12 × 12, and corresponding roots, and use the square and square root keys of a calculator
* recognise and use multiples, factors, primes (less than 100), common factors, highest common factors and lowest common multiples in simple cases
* use simple tests of divisibility
* use the bracket, square, square root and sign change keys of a calculator.
* use the memory of a calculator and interpret the display in different contexts.
 | **Numbers, Powers and Decimals*** add, subtract, multiply and divide positive and negative integers
* round positive numbers to any given power of 10 and decimals to the nearest whole number of one or two decimal places
* use efficient written methods to :add and subtract integers of any size
* multiply and divide 3-digit by 2-digit whole numbers
* use the order of operations, including brackets, with more complex calculations
* multiply and divide integers and decimals by 0.1 or 0.01, and derive products such as 6 × 0.7, 8 × 0.03
* read and write positive integer powers of 10
* use squares, positive and negative square roots, cubes and cube roots, and index notation for small positive integer powers
* use multiples, factors, common factors, highest common factor, lowest common multiple and primes
* find the prime factorisation of a number (e.g. 8000 = 26 × 53)
* use the function keys of a calculator for sign change, brackets, powers and roots, and interpret the display in context.
* use a calculator to carry out more difficult calculations, entering numbers, including fractions, using the memory, and interpreting the display in context
 | **Numbers, Powers and Decimals*** use positive and negative numbers of any size, the laws of arithmetic and inverse operations
* understand the order of operations, including powers
* use rounding to make estimates and to give solutions to an appropriate degree of accuracy
* round decimals to any given number of decimal places or a given number of significant figures
* understand the effects of multiplying or dividing by numbers between 0 and 1
* understand and use equivalences between 0.1, 1⁄10 and 10–1, and multiply and divide by any integer power of 10
* understand the difference between positive and negative square roots and recall cubes of 1, 2, 3, 4, 5 and 10
* use index notation for integer powers
* know and use the index laws for multiplication and division of positive integer powers
* extend mental methods of calculation with factors, powers and roots
* use the power and root keys of a calculator
* use the prime factor decomposition of a number and write the product using index notation
* Find the LCM and HCF of two numbers using Venn diagrams and prime factors
* use a calculator efficiently and appropriately for complex calculations, knowing not to round during intermediate steps
* Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractions and powers of a power
* Use numbers raised to the power zero, including the zero power of 10
* Convert large and small numbers into standard form and vice versa
* Add and subtract numbers in standard form
* Multiply and divide numbers in standard form
* Interpret a calculator display using standard form and know how to enter numbers in standard form
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| **NUMBER 1** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Numbers, Powers and Decimals** * Use index notation for integer powers of 10, including negative powers;
* Recognise powers of 2, 3, 4, 5;
* Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
* Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
* Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
* Use an extended range of calculator functions, including +, –, ×, ÷, *x*², √*x*, memory, *x y*, , brackets;
* Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.
* Identify factors, multiples and prime numbers;
* Find the prime factor decomposition of positive integers – write as a product using index notation;
* Find common factors and common multiples of two numbers;
* Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
* Understand that the prime factor decomposition of a positive integer is unique, whichever factor pair you start with, and that every number can be written as a product of prime factors;
* Convert large and small numbers into standard form and vice versa;
* Add, subtract, multiply and divide numbers in standard form;
* Interpret a calculator display using standard form and know how to enter numbers in standard form;
 | **Powers, Decimals, Indices and Surds*** Recall that *n*0 = 1 and *n*–1 =  for positive integers n as well as,  = √*n* and  = 3√*n* for any positive number *n*;
* Understand that the inverse operation of raising a positive number to a power *n* is raising the result of this operation to the power ;
* Solve problems using HCF and LCM, and prime numbers;
* Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2;
* Simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3).
 | **Powers, Decimals, Indices and Surds*** Solve problems using index laws;
* Find the value of calculations using indices including positive, fractional and negative indices;
 | **Powers, Decimals, Indices and Surds*** understand and use rational and irrational numbers
* use surds and π in exact calculations, without a calculator, and rationalise a denominator such as.
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| **NUMBER 2** |
| **1** | **2** | **3** | **4** |
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 |
| **Fractions and Decimals*** use fraction notation and the vocabulary numerator and denominator
* use unit fractions such as 1⁄2, 1⁄3, 1⁄4, 1⁄5, 1⁄10, … to describe parts of shapes
* recognise simple fractions that are several parts of a whole, such as 2⁄3 or 3⁄4
* Use diagrams to find equivalent fractions or compare fractions
* Write fractions to describe shaded parts of diagrams
* use decimal notation for tenths and hundredths and partition decimals in contexts of money or measurement.
* use decimal notation for tenths and hundredths
* recognise simple equivalent fractions
* express one half, one quarter, three quarters, and tenths and hundredths, as fractions and decimals. recognise proportions of a whole and use simple fractions to describe these
* identify simple equivalent fractions, including relating tenths to hundredths
* find simple equivalent fractions, decimals
* calculate simple fractions of numbers and quantities.
* use efficient written methods to add and subtract decimals
* interpret a calculator display in contexts such as money or measures.
* use decimal notation to record measurements (e.g. 1.25 m or 0.6 kg)
* begin to convert metric units of measurement to related units
* interpret with appropriate accuracy numbers on a range of measuring instruments
* express a quotient as a decimal or fraction
* calculate with simple fractions
* multiply decimals
 | **Fractions and Decimals*** multiply and divide decimals with up to two places by single-digit whole numbers
* understand and use decimal notation and place value
* use the memory of a calculator and interpret the display in different contexts.
* calculate mentally, on paper and with a calculator, as appropriate, to:
* add and subtract simple fractions and those with common denominators
* Express a given number as a fraction of another, using very simple numbers, some cancelling, and where the fraction is both < 1 and > 1
* Write a fraction in its simplest form and find equivalent fractions
* multiply a fraction by an integer
* calculate simple fractions of quantities and measurements.
* convert one metric unit to another (e.g. grams to kilograms)
* use efficient written methods to:
* add and subtract whole numbers and decimals
* use mental methods to multiply and divide simple decimals by one-digit whole numbers, e.g. 0.8 × 6, 2.4 ÷ 3, using jottings as appropriate
* use fractions to describe parts of shapes
* use diagrams to compare two or more simple fractions
* simplify fractions by cancelling and identify equivalent fractions
* recognise equivalent fractions and decimals
 | **Fractions and Decimals*** find equivalent fractions, and equivalent fractions and decimals
* use efficient written methods to:
* add and subtract integers and decimals of any size, including numbers with differing numbers of decimal places
* add and subtract fractions by writing them with a common denominator
* Convert between mixed numbers and improper fractions;
* Add fractions and write the answer as a mixed number
* Multiply and divide an integer by a fraction
* Order fractions, by using a common denominator
* Compare fractions, use inequality signs, compare unit fractions
* Add and subtract mixed number fractions
* calculate fractions of quantities
* use a calculator to carry out more difficult calculations, entering numbers, including fractions, using the memory, and interpreting the display in context
* Recall the fraction-to-decimal conversion
* multiply and divide integers and decimals by 0.1 or 0.01, and derive products such as 6 × 0.7, 8 × 0.03
* use division to convert a fraction to a decimal and recognise that a recurring decimal is a fraction
* order fractions by writing them with a common denominator or by converting them to decimals multiply and divide decimals, understanding where to position the decimal point by considering equivalent calculations
* multiply and divide fractions by integers
* Compare and order fractions, decimals and integers, using inequality signs
 | **Fractions and Decimals*** use efficient written methods to add and subtract decimals of any size, to multiply by decimals, and to divide by decimals by transforming to division by an integer
* Add and subtract mixed number fractions
* Divide mixed numbers by whole numbers and vice versa
* Find the reciprocal of an integer, decimal or fraction
* Understand ‘reciprocal’ as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is not defined)
* Multiply mixed number fractions
* Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions
* Multiply fractions: simplify calculations by cancelling first
* Divide a fraction by a whole number
* Divide fractions by fractions
* Convert between fractions and decimals
* Convert a fraction to a decimal to make a calculation easier, e.g. 0.25 × 8 =  × 8, or  × 10 = 0.375 × 10
* Recognise recurring decimals and convert fractions such as ,  and  into recurring decimals
* use a calculator efficiently and appropriately for complex calculations, knowing not to round during intermediate steps
* know that a recurring decimal is an exact fraction.
* use efficient written methods to add and subtract fractions, and to multiply or divide fractions, interpreting division as a multiplicative inverse, and cancelling common factors before multiplying
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| **NUMBER 2** |
| **5** | **6** | **7** | **8-9** |
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 |
| **Fractions and Decimals*** Add, subtract, multiply and divide decimals, whole numbers including any number between 0 and 1;
* Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
* Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
* Calculate the upper and lower bounds of calculations, particularly when working with measurements;
* Round numbers to the nearest 10, 100, 1000, the nearest integer, to a given number of decimal places and to a given number of significant figures;
* Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.
 | **Fractions and Decimals*** Calculate the upper and lower bounds of an expression involving the four operations;
* Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
* Use inequality notation to specify an error bound.
 | **Fractions and Decimals** * Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
 | **Fractions and Decimals** |
| * Express a given number as a fraction of another;
* Find equivalent fractions and compare the size of fractions;
* Write a fraction in its simplest form, including using it to simplify a calculation, e.g. 50 ÷ 20 =  =  = 2.5;
* Find a fraction of a quantity or measurement, including within a context;
* Convert a fraction to a decimal to make a calculation easier;
* Convert between mixed numbers and improper fractions;
* Add and subtract fractions, including mixed numbers;
* Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;
* Understand and use unit fractions as multiplicative inverses;
 | * Convert a fraction to a recurring decimal and vice versa;
* Find the reciprocal of an integer, decimal or fraction;
 | * By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
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GCSE ALGEBRA

*Threshold concepts for 5 yr Maths Course (Grades 1-9)*

*Version 29/05/17*

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 |
| **Expressions, Equations and Formulae*** Begin to use letters and symbols to represent numbers
* Write simple expressions
* simplify simple linear expressions by collecting like terms
* use simple formulae expressed in words
* use simple formulae expressed in words, then symbols
* substitute numbers into word formulae
* substitute positive integers into simple linear expressions and formulae
* solve simple linear equations using an appropriate method, e.g. 4*a* = 12.
 | **Expressions, Equations and Formulae*** use notation and symbols correctly
* write an expression
* understand the difference between expression, equation and formula
* simplify linear algebraic expressions by collecting like terms
* understand that algebraic operations follow the rules of arithmetic
* multiply together simple expressions
* use index notation when multiplying and algebraic terms
* multiply a constant over a bracket (integer coefficients)
* substitute positive integers into linear expressions and formulae
* use simple formulae from mathematics and other subjects.
* derive a formula
* construct and solve simple linear equations with integer coefficients (unknown on one side only)
 | **Expressions, Equations and Formulae*** select an expression, equation, formula or identity from a list
* use index laws in algebra
* understand the symbol not equal to and the identity sign
* simplify expressions involving brackets
* multiply a single term over a bracket
* recognise factors of algebraic terms involving single brackets
* construct and solve linear equations with integer coefficients (unknown on one or both sides, without and with brackets), e.g. by using inverse operations or by transforming both sides in the same way
* substitute positive and negative numbers into expressions
* derive and substitute integers into simple formulae and expressions, including examples that lead to an equation to solve
* Rearrange simple equations
* Solve angle or perimeter problems using algebra
 | **Expressions, Equations and Formulae*** represent and interpret problems in algebraic or graphical form
* use index laws in algebra
* argue mathematically to show expressions are equivalent
* factorise algebraic expressions by taking out single-term common factors
* substitute numbers into expressions and formulae and, in simple cases, change the subject of a formula
* construct and solve linear equations with integer coefficients (with and without brackets, negative signs anywhere in the equation, positive or negative solution)
* Solve linear equations in one unknown, with integer or fractional coefficients;
* Solve angle or perimeter problems using algebra
* find the inverse of a linear function
* use algebraic methods to solve problems involving direct proportion, relating solutions to graphs of the equations, using ICT as appropriate
 |
| **Sequences*** recognise, describe and extend simple number sequences
* recognise sequences of odd and even numbers and other sequences including Fibonacci
* use function machines to find terms of a sequence
* write term-to-term definition of a sequence in words
* Recognise such sequences from diagrams and draw the next term in a pattern sequence
 | **Sequences*** describe integer sequences using the term to term definition
* find a term in a sequence using the term to term rule
* generate the terms of a simple sequence, given a rule (e.g. finding a term from the previous term, finding a term given its position in the sequence)
* generate sequences from patterns or practical contexts and describe the general term in simple cases.
* generate the terms of a simple sequence, given a rule
* generate sequences from patterns and describe the general term in simple cases
* represent simple functions using words, symbols and mappings
* recognise the first few triangular numbers, square and cube numbers
* find the nth term of a pattern sequence
 | **Sequences*** generate terms of a linear sequence using term-to-term and position-to-term rules, on paper and using a spreadsheet or graphics calculator
* find the nth term of arithmetic and linear sequences
* find a specific term in the sequence using the nth term
* use linear expressions to describe the nth term of a simple arithmetic sequence, justifying its form by referring to the context from which it was generated.
* Use the nth term to decide if a given number is a term in the sequence
 | **Sequences*** generate terms of a sequence using term-to-term and position-to-term rules, on paper and using ICT
* generate sequences from practical contexts and write and justify an expression for the nth term of an arithmetic sequence
* Use the *n*th term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term over a certain number
* Use the *n*th term of an arithmetic sequence to find the first term greater/less than a certain number
* Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms;
* Continue a quadratic sequence and use the nth term to generate terms;
 |
| **Inequalities*** Show inequalities on number lines;
* Write down whole number values that satisfy an inequality
 | **Inequalities*** Show inequalities on number lines;
* Write down whole number values that satisfy an inequality
 | **Inequalities*** Solve an inequality such as –3 < 2x + 1 <7 and show the solution set on a number line;
* Solve two inequalities in x, find the solution sets and compare them to see which value of x satisfies both;
* Use the correct notation to show inclusive and exclusive inequalities;
* Construct inequalities to represent a set shown on a number line;
 | **Inequalities*** Use the correct notation to show inclusive and exclusive inequalities;
* Construct inequalities to represent a set shown on a number line;
* Solve simple linear inequalities in one variable, and represent the solution set on a number line;
* Round answers to a given degree of accuracy
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| **ALGEBRA 2** |
| **5** | **6** | **7** | **8-9** |
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| **Functions and Graphs*** Identify and plot points in all four quadrants;
* Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
* Draw distance–time and velocity–time graphs;
* Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
* Find the coordinates of the midpoint of a line segment from coordinates;
* Find the equation of the line through two given points.
* Plot and draw graphs of *y* = *a*, *x* = *a*, *y* = *x* and *y* = –*x*, drawing and recognising lines parallel to axes, plus *y* = *x* and *y* = –*x*;
* Identify and interpret the gradient of a line segment;
* Recognise that equations of the form *y* = *mx* + *c* correspond to straight-line graphs in the coordinate plane;
* Identify and interpret the gradient and *y*-intercept of a linear graph given by equations of the form *y* = *mx* + *c*;
* Find the equation of a straight line from a graph in the form *y* = *mx* + *c*;
* Plot and draw graphs of straight lines of the form *y* = *mx* + *c* with and without a table of values;
* Find the equation of the line through one point with a given gradient;
* find approximate solutions to a linear equation from a graph;
* identify direct proportion from a graph;
 | **Functions and Graphs*** Calculate the length of a line segment given the coordinates of the end points;
* Find the coordinates of points identified by geometrical information.
* Identify and interpret gradient from an equation *ax* + *by* = *c*;
* Find the equation of a straight line from a graph in the form *ax* + *by* = *c*;
* Plot and draw graphs of straight lines in the form *ax* + *by* = *c*;
* Interpret and analyse information presented in a range of linear graphs:
* use gradients to interpret how one variable changes in relation to another;
* Select and use the fact that when *y* = *mx* + *c* is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of *m* and a line perpendicular to this line will have a gradient of .
 | **Functions and Graphs*** find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
* Explore the gradients of parallel lines and lines perpendicular to each other;
* Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
* Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
* Use function notation;
* Find f(*x*) + g(*x*) and f(*x*) – g(*x*), 2f(*x*), f(3*x*) etc algebraically;
* Find the inverse of a linear function;
* Know that f –1(*x*) refers to the inverse function;
* For two functions f(*x*) and g(*x*), find gf(*x*).
 | **Functions and Graphs*** Find the equation of a tangent to a circle at a given point, by:
* finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
* finding the gradient of the tangent perpendicular to it;
* using the given point;
* Recognise and construct the graph of a circle using *x*2 + *y*2 = *r*2 for radius *r* centred at the origin of coordinates.
 |
| **Other Graphs*** Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
* Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
* Draw graphs of simple cubic functions using tables of values;
* Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
* Draw graphs of the reciprocal function  with *x* ≠ 0 using tables of values;
* Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
 | **Other Graphs*** Interpret graphs of quadratic functions from real-life problems;
* Interpret the rate of change of graphs of containers filling and emptying;
* Interpret the rate of change of unit price in price graphs.
* Interpret the gradient of a linear or non-linear graph in financial contexts;
 | **Other Graphs*** Draw circles, centre the origin, equation *x*2 + *y*2 = *r*2.
* Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of f(*x*) + *a*, or f(*x* – *a*):
* apply to the graph of *y* = f(*x*) the transformations *y* = –f(*x*), *y* = f(–*x*) for linear, quadratic, cubic functions;
* apply to the graph of y = f(*x*) the transformations *y* = f(*x*) + *a*, *y* = f(*x* + *a*) for linear, quadratic, cubic functions;
 | **Other Graphs*** Estimate area under a quadratic or other graph by dividing it into trapezia;
* Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
* Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
* for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
* for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
* Interpret the area under a linear or non-linear graph in real-life contexts;
 |
| **Quadratic & Simultaneous Equations*** solve quadratic equations of the form *x*2 + *b x* + *c* = 0 by factorisation.
* Find approximate solutions to quadratic equations using a graph;
 | **Quadratic & Simultaneous Equations*** Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;
* Factorise quadratic expressions in the form *ax*2 + *bx* + *c*;
* Set up and solve quadratic equations;
* Solve quadratic equations by factorisation and completing the square;
* Solve quadratic equations by using the quadratic formula;
 | **Quadratic & Simultaneous Equations*** solve quadratic equations and inequalities by factorisation, completing the square and using the quadratic formula, including those in which the coefficient of the quadratic term is greater than 1
* Use iteration with simple converging sequences.
* Solve quadratic equations that need rearranging;
 | **Quadratic & Simultaneous Equations*** Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots and *y*-intercept, turning point;
* Be able to identify from a graph if a quadratic equation has any real roots;
 |
| * Find the exact solutions of two simultaneous equations in two unknowns;
* Use elimination or substitution to solve simultaneous equations;
* Solve simultaneous equations graphically:
 | * find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
* solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
 | * Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:

linear / linear, including where both need multiplying;linear / quadratic;linear / *x*2 + *y*2 = *r*2;* find graphically the intersection points of a given straight line with a circle;
 | * consider cases of simultaneous linear equations that have no solution or an infinite number of solutions.
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GCSE GEOMETRY

*Threshold concepts for 5 yr Maths Course (Grades 1-9)*

*Version 29/05/17*

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| **GEOMETRY 1** |
| **1** | **2** | **3** | **4** |
| Students should be able to demonstrate fluency in functional mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Limited Confidence*** | **Fluency - Increasing ability to use and apply standard techniques** Grade 2 Students should, for Grade 1 and functional mathematical topics, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts** Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Angles and Constructions*** estimate and measure length
* interpret with appropriate accuracy numbers on a range of measuring instruments
* measure and draw lines to the nearest millimetre
* describe angles as turns and use degrees
* understand clockwise and anticlockwise
* Know that there are 360° in a full turn, 180° in a half turn and 90° in a quarter turn;
* Identify a line perpendicular to a given line;
* Mark perpendicular lines on a diagram and use their properties
* Identify parallel lines
* estimate, measure and draw acute and obtuse angles
* calculate angles on a straight line or at a point
* begin to calculate angles in a triangle
* Use the fact that angle sum of a quadrilateral is 360
* Draw circles and arcs to a given radius or diameter
* Draw sketches of shapes
* Use isometric grids to draw 2D representations of 3D solids
* Know and use compass directions
 | **Angles and Constructions*** draw accurate diagrams and constructions
* use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes
* distinguish between and estimate acute, obtuse and reflex angles and measure and draw them to the nearest degree
* know and calculate the sum of angles at a point and on a straight line, and recognise vertically opposite angles
* use correctly the vocabulary, notation and labelling conventions for lines, angles and shapes
* calculate the sum of angles at a point, on a straight line and in a triangle, and recognise vertically opposite angles
* use a ruler, set square and protractor to:

-estimate, measure and draw lines to the nearest millimetre and acute, obtuse and reflex angles to the nearest degree-draw parallel and perpendicular lines-construct squares and rectangles* Distinguish between scalene, equilateral, isosceles and right-angled triangles
* construct a triangle given two sides and the included angle (SAS) or two angles and the included side (ASA)
* draw simple nets of 3D shapes
* Understand and draw plans and elevations and be able to draw a solid from their 2D representation
 | **Angles, Bearings and Constructions*** identify alternate angles and corresponding angles
* Understand and use the angle properties of intersecting lines
* Recall and use properties of angles at a point, angles at a point on a straight line, right angles, and vertically opposite angles
* understand a proof that:
* the angle sum of a triangle is 180° and of a quadrilateral is 360°
* Understand and use the angle properties of triangles, use the symmetry property of isosceles triangle to show that base angles are equal;
* Use the side/angle properties of isosceles and equilateral triangles
* the exterior angle of a triangle is equal to the sum of the two interior opposite angles.
* Understand and use the angle properties of quadrilaterals
* use straight edge and compasses to construct:

-the midpoint and perpendicular bisector of a line segment-the bisector of an angle-the perpendicular from a point to a line* use ruler and compasses to construct a triangle, given the lengths of the three sides (SSS)
* Draw and construct diagrams from given instructions, including the following:
* a region bounded by a circle and an intersecting line;
* a given distance from a point and a given distance from a line;
* equal distances from two points or two line segments;
* regions may be defined by ‘nearer to’ or ‘greater than’
* make scale drawings
* Mark on a diagram the position of point *B* given its bearing from point *A*
* use bearings to specify direction Mark on a diagram the position of point *B* given its bearing from point *A*;
* find simple loci, both by reasoning and by using ICT, to produce shapes and paths.
* Understand and draw plans and elevations and be able to draw a solid from their 2D representation
 | **Angles, Bearings and Constructions*** Use the fact that angle sum of a quadrilateral is 360°
* Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices
* Understand and use the angle properties of parallel lines
* explain how to find, calculate and use:

-the sums of the interior and exterior angles of quadrilaterals, pentagons and hexagons-the interior and exterior angles of regular polygons Use the sum of angles of irregular polygons; * Calculate and use the sums of the interior angles of polygons;
* Calculate and use the angles of regular polygons;
* Use the sum of the interior angles of an *n*-sided polygon; Use the sum of the exterior angles of any polygon is 360°;
* Use the sum of the interior angle and the exterior angle is 180
* use straight-edge and compasses to construct:

-the perpendicular from a point to a line-the perpendicular to a line from a point on the line-triangles, given right angle, hypotenuse and side (RHS) * Given the bearing of a point *A* from point *B*, work out the bearing of *B* from *A*;
* Use accurate drawing to solve bearings problems;
* Estimate lengths using a scale diagram
* Make an accurate scale drawing from a diagram
* Give a bearing between the points on a map or scaled plan
* Solve locus problems including bearings.
* find the locus of a point that moves according to a simple rule, both by reasoning and by using ICT
* use ICT to explore constructions of triangles and other 2D shapes
* Find and describe regions satisfying a combination of loci
* Use constructions to solve loci problems (2D only)
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| **GEOMETRY 1** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Angles, Loci and Trigonometry*** Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
* Understand ‘regular’ and ‘irregular’ as applied to polygons;
* Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle;
* Use symmetry property of an isosceles triangle to show that base angles are equal;
* Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
* Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
* Explain why the angle sum of a quadrilateral is 360°; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is 360°;
* Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
* Use the angle sums of irregular polygons;
* Calculate and use the sums of the interior angles of polygons, use the sum of angles in a triangle to deduce and use the angle sum in any polygon and to derive the properties of regular polygons;
* Use the sum of the exterior angles of any polygon is 360°;
* Use the sum of the interior angles of an n-sided polygon;
* Use the sum of the interior angle and the exterior angle is 180°;
* Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons;
* Understand, draw and measure bearings;
* Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
* Use the standard ruler and compass constructions:
* bisect a given angle;
* construct a perpendicular to a given line from/at a given point;
* construct angles of 90°, 45°;
* perpendicular bisector of a line segment;
* Construct:
* a region bounded by a circle and an intersecting line;
* a given distance from a point and a given distance from a line;
* equal distances from two points or two line segments;
* regions which may be defined by ‘nearer to’ or ‘greater than’;
 | **Angles, Loci and Trigonometry*** Calculate the angles of regular polygons and use these to solve problems;
* Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
* Use angle facts to demonstrate how shapes would ‘fit together’, and work out interior angles of shapes in a pattern.
* Use constructions to solve loci problems including with bearings;
* Know that the perpendicular distance from a point to a line is the shortest distance to the line.
 | **Angles, Loci and Trigonometry*** Find and describe regions satisfying a combination of loci, including in 3D;
 | **Angles, Loci and Trigonometry*** Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
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| **GEOMETRY 2** |
| **1** | **2** | **3** | **4** |
| Students should be able to demonstrate fluency in functional mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Limited Confidence*** | **Fluency - Increasing ability to use and apply standard techniques** Grade 2 Students should, for Grade 1 and functional mathematical topics, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts** Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Transformations and Vectors*** Understand clockwise and anticlockwise
* recognise shapes with no lines of symmetry.
* identify lines of symmetry in 2D shapes
* visualise and draw where a shape will be after reflection in a mirror line
* Understand that reflections are specified by a mirror line
* Identify correct reflections from a choice of diagrams
* explore symmetry and simple transformations using ICT.
* make and describe turns, e.g. from SW to N
* Draw the position of a shape after rotation about a centre (not on a coordinate grid)
* Identify correct rotations from a choice of diagrams
 | **Transformations and Vectors*** recognise and visualise the symmetries of a 2D shape
* Transform 2D shapes using single reflections (including those not on coordinate grids) with vertical, horizontal and diagonal mirror lines
* Describe reflections on a coordinate grid
* Understand that rotations are specified by a centre, an angle and a direction of rotation
* Rotate a shape about the origin or any other point on a coordinate grid
* Understand that translations are specified by a distance and direction using a vector
* Translate a given shape by a vector
* Describe and transform 2D shapes using single translations on a coordinate grid
 | **Transformations and Vectors*** identify all the symmetries of 2D shapes
* Identify the equation of a line of symmetry
* Scale a shape on a grid (without a centre specified)
* Find the centre of rotation, angle and direction of rotation and describe rotations
* Describe a rotation fully using the angle, direction of turn, and centre
* transform 2D shapes by rotation, reflection and translation, on paper and using ICT, and try out mathematical representations of simple combinations of these transformations
* enlarge 2D shapes, given a centre of enlargement and a positive integer scale factor, and explore enlargement using ICT.
* Understand that an enlargement is specified by a centre and a scale factor
* Enlarge a given shape using (0, 0) as the centre of enlargement, and enlarge shapes with a centre other than (0, 0)
* Describe and transform 2D shapes using enlargements by a positive integer scale factor
* Use column vectors to describe translations
* Understand that distances and angles are preserved under rotations and translations, so that any figure is congruent under either of these transformations
* Understand and use column notation in relation to vectors
 | **Transformations and Vectors*** identify reflection symmetry in 3D shapes
* use a coordinate grid to solve problems involving translations, rotations, reflections and enlargements
* recognise that translations, rotations and reflections preserve length and angle, and map objects onto congruent images
* explore and compare combinations of translations, reflections and rotations of 2D shapes, on paper and using ICT
* enlarge 2D shapes, given a centre of enlargement and a positive integer scale factor, identifying the scale factor as the ratio of the lengths of any two corresponding line segments
* recognise that enlargements preserve angle but not length
* Find the centre of enlargement by drawing
* Describe and transform 2D shapes using enlargements by a fractional scale factor
* Understand that distances and angles are preserved under reflections, so that any figure is congruent under this transformation
* Understand that similar shapes are enlargements of each other and angles are preserved – define similar in this unit
* Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements
* Be able to represent information graphically given column vectors
* Identify two column vectors which are parallel
* Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector
 |
| **Properties of 2D and 3D shapes*** identify lines of symmetry in 2D shapes
* Know the terms face, edge and vertex
* Identify parallel lines
* Identify a line perpendicular to a given line
* identify, visualise and describe properties of rectangles, triangles, regular polygons and 3D solids, including parallel and perpendicular faces or edges
* visualise and make 3D mathematical models by linking faces or edges
* identify and draw nets of simple 3D shapes
* List the properties of each special type of quadrilateral, or identify (name) a given shape
* Draw sketches of shapes
* Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone
 | **Properties of 2D and 3D shapes*** Recall the properties and definitions of special types of quadrilaterals, including symmetry properties
* visualise 3D shapes and deduce some of their properties
* Mark perpendicular lines on a diagram and use their properties
* use coordinates in the first quadrant to draw, locate and complete shapes with given properties
* Mark parallel lines on a diagram and use their properties
* Name all quadrilaterals that have a specific property
* Identify quadrilaterals from everyday usage
* Classify quadrilaterals by their geometric properties
* Distinguish between scalene, equilateral, isosceles and right-angled triangles
 | **Properties of 2D and 3D shapes*** classify quadrilaterals by their geometrical properties
* identify all the symmetries of 2D shapes
* Recognise and name pentagons, hexagons, heptagons, octagons and decagons
* Understand ‘regular’ and ‘irregular’ as applied to polygons
* Identify shapes which are congruent (by eye)
 | **Pythagoras and Trigonometry*** Understand, recall and use Pythagoras’ Theorem in 2D, including leaving answers in surd form
* Given 3 sides of a triangle, justify if it is right-angled or not
* Calculate the length of the hypotenuse in a right-angled triangle, including decimal lengths and a range of units
* Find the length of a shorter side in a right-angled triangle
* Apply Pythagoras’ Theorem with a triangle drawn on a coordinate grid
* Calculate the length of a line segment AB given pairs of points
* Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures
* Use the trigonometric ratios to solve 2D problems
* Find angles of elevation and depression
* Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question
* Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.
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| **GEOMETRY 2** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Transformations and Vectors*** Distinguish properties that are preserved under particular transformations;
* Recognise and describe rotations – know that that they are specified by a centre and an angle;
* Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
* Identify the equation of a line of symmetry;
* Recognise and describe reflections on a coordinate grid – know to include the mirror line as a simple algebraic equation, *x* = *a*, *y* = *a*, *y* = *x*, *y* = –*x* and lines not parallel to the axes;
* Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also *y* = *x* and *y* = –*x*;
* Recognise and describe single translations using column vectors on a coordinate grid;
* Translate a given shape by a vector;
* Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
* Enlarge a shape on a grid without a centre specified;
* Know that an enlargement on a grid is specified by a centre and a scale factor;
* Identify the scale factor of an enlargement of a shape;
* Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing (include fractional sf);
* Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
* Understand that 2**a** is parallel to **a** and twice its length, and that **a** is parallel to –**a** in the opposite direction.
* Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
* Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
 | **Transformations and Vectors*** Describe the changes and invariance achieved by combinations of rotations, reflections and translations.
* Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
* Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
* Find the length of a vector using Pythagoras’ Theorem.
* Calculate the resultant of two vectors.
* Solve geometric problems in 2D where vectors are divided in a given ratio.
 | **Transformations and Vectors*** Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.
 | **Transformations and Vectors** |
| **Pythagoras and Trigonometry*** Understand, recall and use Pythagoras’ Theorem in 2D;
* Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
* Find the length of a shorter side in a right-angled triangle;
* Calculate the length of a line segment *AB* given pairs of points;
* Give an answer to the use of Pythagoras’ Theorem in surd form;
* Use the trigonometric ratios to solve 2D problems;
* Know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45°, 60° and 90°; know the exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60°.
* Know the exact values of sin *θ* and cos *θ* for *θ* = 0°, 30°, 45° , 60° and 90° and exact value of tan *θ* for *θ* = 0°, 30°, 45° and 60° and find them from graphs.
 | **Pythagoras and Trigonometry*** understand and use trigonometric relationships in right-angled triangles, and use these to solve problems, including those involving bearings
* Understand the language of planes, and recognise the diagonals of a cuboid.
* Find angles of elevation and depression;
* Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
* Given three sides of a triangle, justify if it is right-angled or not;
 | **Pythagoras and Trigonometry*** Solve geometrical problems on coordinate axes.
* Understand, recall and use trigonometric relationships and Pythagoras’ Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
* Calculate the length of a diagonal of a cuboid.
	+ Find the angle between a line and a plane.
* Know and apply Area = *ab* sin *C* to calculate the area, sides or angles of any triangle.
* Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
* Use the sine and cosine rules to solve 3D problems.
* Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) *y* = sin *x*, *y* = cos *x* and *y* = tan *x* for angles of any size.
 | **Pythagoras and Trigonometry*** Apply to the graph of *y* = f(*x*) the transformations *y* = –f(*x*), *y* = f(–*x*) for sine, cosine and tan functions f(*x*).
* Apply to the graph of *y* = f(*x*) the transformations *y* = f(*x*) + *a*, *y* = f(*x* + *a*) for sine, cosine and tan functions f(*x*).
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| **GEOMETRY 3** |
| **1** | **2** | **3** | **4** |
| Students should be able to demonstrate fluency in functional mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Limited Confidence*** | **Fluency - Increasing ability to use and apply standard techniques** Grade 2 Students should, for Grade 1 and functional mathematical topics, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts** Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Length, Perimeter, Area and Volume*** Indicate given values on a scale, including decimal value
* Know that measurements using real numbers depend upon the choice of unit
* Convert between units of measure within one system
* measure and calculate the perimeter of rectangles and triangles
* estimate areas by counting squares
* derive and use the formula for the area of a rectangle
* Recall the definition of a circle
 | **Length, Perimeter, Area and Volume*** Convert metric units to metric units
* Make sensible estimates of a range of measures in everyday settings
* know and use the formula for the area of a rectangle
* Find the perimeter of parallelograms and trapezia
* Recall and use the formulae for the area of a triangle and rectangle
* Find the area of a rectangle and triangle
* calculate perimeters and areas of shapes made from rectangles
* visualise 3D shapes and deduce some of their properties
* calculate the surface areas of cubes and cuboids
* Identify, name and draw parts of a circle including tangent, chord and segment
 | **Length, Perimeter, Area and Volume*** derive and use formulae for the area of a triangle, parallelogram and trapezium and the volume of a cuboid
* calculate areas of compound shapes and volumes and surface areas of cuboids and shapes made from cuboids
* Find the area of a trapezium and recall the formula
* Find the area of a parallelogram
* Calculate areas and perimeters of compound shapes made from triangles and rectangles
* Find surface area using rectangles and triangles
* Recall and use the formula for the volume of a cuboid
* Recall and use formulae for the circumference of a circle and the area enclosed by a circle circumference of a circle = 2πr = πd, area of a circle = πr2
* Find circumferences and areas enclosed by circles
 | **Length, Perimeter, Area and Volume*** know and use the formulae for the circumference and area of a circle, and use the π key of a calculator
* Find circumferences and areas enclosed by circles
* Give an answer to a question involving the circumference or area of a circle in terms of π
* Find radius or diameter, given area or perimeter of a circles
* Find the perimeters and areas of semicircles and quarter-circles
* Calculate perimeters and areas of composite shapes made from circles and parts of circles
* Calculate arc lengths, angles and areas of sectors of circles
* Find the surface area of a cylinder
* Find the volume of a cylinder
* Find the surface area and volume of spheres, pyramids, cones and composite solids
* calculate the surface area and volume of right prisms.
* Find the volume of a prism, including a triangular prism, cube and cuboid
* Estimate surface areas by rounding measurements to 1 significant figure
* Convert between metric area measures
* Estimate volumes etc by rounding measurements to 1 significant figure
* Convert between metric volume measures
* Convert between metric measures of volume and capacity e.g. 1ml = 1cm3
 |
| **Geometrical Reasoning** | **Geometrical Reasoning** | **Geometrical Reasoning*** solve geometrical problems using side and angle properties of triangles and special quadrilaterals, explaining reasoning with diagrams and text
* Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS)
* Identify shapes which are similar; including all circles or all regular polygons with equal number of sides
 | **Geometrical Reasoning*** represent problems in geometric form, making accurate mathematical diagrams on paper and on screen
* Solve angle problems involving congruence
* Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity
* Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides
* Understand the effect of enlargement on perimeter of shapes
* Solve problems to find missing lengths in similar shapes
* Know that scale diagrams, including bearings and maps are ‘similar’ to the real-life examples
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| **GEOMETRY 3** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Length, Perimeter, Area and Volume*** use, convert and calculate using metric and, where appropriate, imperial measures
* consolidate analysing 3D shapes through 2D projections, including plans and elevations
* solve problems involving surface areas and volumes of cylinders, spheres, cones and composite solids.
* solve problems involving lengths of circular arcs and areas of sectors
* Find the volume and surface area of a cylinder;
* Use the formulae for volume and surface area of spheres and cones;
* Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
* Draw 3D shapes using isometric grids;
* Understand and draw front and side elevations and plans of shapes made from simple solids;
* Calculate arc lengths, angles and areas of sectors of circles;
* Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures;
* Calculate the area of compound shapes made from triangles, rectangles, trapezia and parallelograms using a variety of metric measures;
* Find the perimeter of a rectangle, trapezium and parallelogram using a variety of metric measures;
* Calculate the perimeter of compound shapes made from triangles and rectangles;
* Estimate area and perimeter by rounding measurements to 1 significant figure to check reasonableness of answers;
* Recall the definition of a circle and name and draw parts of a circle;
* Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference = 2*πr* = *πd* and area of a circle = *πr*2) using a variety of metric measures;
* Use *π* ≈ 3.142 or use the *π* button on a calculator;
* Find radius or diameter, given area or circumference of circles in a variety of metric measures;
* Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes with and without a diagram;
* Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm3;
* Estimating surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers;
* Use *π* ≈ 3.142 or use the *π* button on a calculator;
 | **Length, Perimeter, Area and Volume*** Recall and use the formula for volume of pyramid;
* Find the surface area of a pyramid;
* Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
* Give answers in terms of *π*;
* Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
 | **Length, Perimeter, Area and Volume*** Form equations involving more complex shapes and solve these equations.
* Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
 | **Length, Perimeter, Area and Volume** |
| **Geometrical Reasoning*** Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
* Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
* Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
* Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
* Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
* Find missing lengths, areas and volumes in similar 3D solids;
* Use the relationship between enlargement and areas and volumes of simple shapes and solids;
 | **Geometrical Reasoning*** Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
* Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;
* Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
* Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
* Solve angle problems by first proving congruence;
 | **Geometrical Reasoning*** Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.
* Use formal geometric proof for the similarity of two given triangles;
* Prove and use the facts that:
* the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
* the angle in a semicircle is a right angle;
* the perpendicular from the centre of a circle to a chord bisects the chord;
* angles in the same segment are equal;
* alternate segment theorem;
* opposite angles of a cyclic quadrilateral sum to 180°;
* Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
* Find and give reasons for missing angles on diagrams using:
* circle theorems;
* isosceles triangles (radius properties) in circles;
* the fact that the angle between a tangent and radius is 90°;
* the fact that tangents from an external point are equal in length.
 | **Geometrical Reasoning** |

GCSE PROBABILITY

*Threshold concepts for 5 yr Maths Course (Grades 1-9)*

*Version 29/05/17*

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| **PROBABILITY 1** |
| **1** | **2** | **3** | **4** |
| Students should be able to demonstrate fluency in functional mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Limited Confidence*** | **Fluency - Increasing ability to use and apply standard techniques** Grade 2 Students should, for Grade 1 and functional mathematical topics, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts** Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Probability*** discuss events using words such as ‘likely’, ‘unlikely’, ‘certain’, ‘impossible’.
* place the probability of events on a scale from impossible to certain.
* use the language of chance or likelihood
* find probabilities based on equally likely outcomes in simple contexts.
* List all outcomes for single events systematically
 | **Probability*** use the vocabulary and ideas of probability, drawing on experience
* understand and use the probability scale from 0 to 1
* find and justify probabilities based on equally likely outcomes in simple contexts
* identify all the possible mutually exclusive outcomes of a single event.
* Find the probability of an event happening using theoretical probability
* Use theoretical models to include outcomes using dice, spinners, coins
* Write probabilities in words or fractions, decimals and percentages
* Work out probabilities from frequency tables
* Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1
 | **Probability*** interpret results of an experiment using the language of probability and appreciate that random processes are unpredictable
* know that, if the probability of an event occurring is *p*, then the probability of it not occurring is 1 – *p*
* use diagrams and tables to record all possible mutually exclusive outcomes for single events and for two successive events
* Work out probabilities from two-way tables
* Record outcomes of probability experiments in tables
* Add simple probabilities
* List all outcomes for combined events systematically
* Use and draw sample space diagrams
 | **Probability** * identify all the mutually exclusive outcomes of an experiment
* know that the sum of probabilities of all mutually exclusive outcomes is 1 and use this when solving problems
* use a numerical scale from 0 to 1 to express and compare experimental and theoretical probabilities in a range of contexts.
* Find a missing probability from a list or table including algebraic terms
 |
| **Probability 2 and Venn Diagrams** | **Probability 2 and Venn Diagrams*** estimate probabilities by collecting data from a simple experiment and recording in a frequency table
* compare experimental and theoretical probabilities in simple contexts.
* Find the probability of an event happening using relative frequency
* Estimate the number of times an event will occur, given the probability and the number of trials – for both experimental and theoretical probabilities
* List all outcomes for combined events systematically
 | **Probability 2 and Venn Diagrams*** compare estimated experimental probabilities with theoretical probabilities, recognising that:
* if an experiment is repeated, the outcome may and usually will be different
* increasing the number of times an experiment is repeated generally leads to better estimates of probability
* Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values
* Compare experimental data and theoretical probabilities
* Use tree diagrams to calculate the probability of two independent events
 | **Probability 2 and Venn Diagrams*** use a numerical scale from 0 to 1 to express and compare experimental and theoretical probabilities in a range of contexts.
* appreciate the difference between mathematical explanation and experimental evidence.
* Use union and intersection notation
* Compare relative frequencies from samples of different sizes
* Find the probability of successive events, such as several throws of a single dice
* Use tree diagrams to calculate the probability of two dependent events
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| **PROBABILITY 1** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Probability*** Write probabilities using fractions, percentages or decimals;
* Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;
* Estimate the number of times an event will occur, given the probability and the number of trials;
* Find the probability of successive events, such as several throws of a single dice;
* List all outcomes for single events, and combined events, systematically;
* Draw sample space diagrams and use them for adding simple probabilities;
* Know that the sum of the probabilities of all outcomes is 1;
* Use 1 – *p* as the probability of an event not occurring where *p* is the probability of the event occurring;
* Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
 | **Probability*** Use the product rule for counting (i.e. if there are *m* ways of doing one task and for each of these, there are *n* ways of doing another task, then the total number of ways the two tasks can be done is *m* × *n* ways);
* Find a missing probability from a list or two-way table, including algebraic terms;
* Understand conditional probabilities and decide if two events are independent;
* Use a two-way table to calculate conditional probability;
* Compare experimental data and theoretical probabilities;
* Compare relative frequencies from samples of different sizes.
 | **Probability*** Understand selection with or without replacement;
* Use a tree diagram to calculate conditional probability;
* Calculate the probability of independent and dependent combined events;
 | **Probability** |
| **Probability 2 and Venn Diagrams*** Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values;
* Use union and intersection notation;
 | **Probability 2 and Venn Diagrams** | **Probability 2 and Venn Diagrams** | **Probability 2 and Venn Diagrams*** Use a Venn diagram to calculate conditional probability;
 |

GCSE RATIO & PROPORTION

*Threshold concepts for 5 yr Maths Course (Grades 1-9)*

*Version 29/05/17*

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| **RATIO AND PROPORTION 1** |
| **1** | **2** | **3** | **4** |
| Students should be able to demonstrate fluency in functional mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Limited Confidence*** | **Fluency - Increasing ability to use and apply standard techniques** Grade 2 Students should, for Grade 1 and functional mathematical topics, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts** Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Ratio and Proportion*** recognise proportions of a whole and use simple fractions and percentages to describe these
* find equivalent fractions, decimals and percentages
* multiply a fraction by an integer
* calculate simple fractions and percentages of numbers and quantities
* solve simple problems involving ideas of direct proportion by scaling numbers up or down.
* Write ratios in their simplest form
* Write a ratio as a fraction
* Convert between currencies
* Find amounts for 3 people when amount for 1 given
 | **Ratio and Proportion*** Understand and express the division of a quantity into a of number parts as a ratio
* Work out which product is the better buy
* express a smaller number as a fraction or percentage of a larger one
* use percentages to compare simple proportions
* calculate simple fractions and percentages of quantities and measurements
* Scale up recipes
* understand the relationship between ratio and proportion
* use direct proportion in simple contexts
* Solve proportion problems using the unitary method
* use ratio notation, simplify ratios and divide a quantity into two parts in a given ratio
* Write/interpret a ratio to describe a situation
* solve simple problems involving ratio and direct proportion using informal strategies.
* find equivalent percentages, fractions and decimals
* calculate with whole numbers, decimals, fractions and percentages
* Compare ratios
* Express a multiplicative relationship between two quantities as a ratio or a fraction
 | **Ratio and Proportion*** apply understanding of the relationship between ratio and proportion
* Understand and use proportion as equality of ratios
* Solve word problems involving direct proportion
* simplify ratios, including ratios expressed in different units, recognising links with fraction notation
* divide a quantity into two or more parts in a given ratio
* calculate fractions of quantities, using a calculator where appropriate
* Recognise when values are in direct proportion by reference to the graph form
* use equivalent fractions, decimals and percentages to compare proportions
* use the unitary method to solve problems involving ratio and direct proportion
* Write ratios in form 1 : m or m : 1
* Share a quantity in a given ratio including three-part ratios
* Solve a ratio problem in context:
* use a ratio to find one quantity when the other is known
* use a ratio to compare a scale model to a real-life object
* use a ratio to convert between measures and currencies
* problems involving mixing, e.g. paint colours, cement and drawn conclusions
 | **Ratio and Proportion*** recognise when fractions or percentages are needed to compare proportions
* use proportional reasoning to solve problems, choosing the correct numbers to take as 100%, or as a whole
* compare two ratios and calculate ratios in a range of contexts.
* Write a ratio as a linear function
* Write lengths, areas and volumes of two shapes as ratios in simplest form
* Write ratios in form 1 : m or m : 1
* Solve word problems involving direct and indirect proportion
* Understand inverse proportion: as x increases, y decreases (inverse graphs done in later unit)
* Understand direct proportion ---> relationship y = kx
 |
| **Compound Measures*** read values in km/h and mph from a speedometer
* calculate average speed, distance, time – in miles per hour as well as metric measures
 | **Compound Measures*** Understand and use compound measures
* Density
* Pressure
* Speed
 | **Compound Measures*** convert between metric speed measures
* Express a given number as a percentage of another number in more complex situations
* Calculate percentage profit or loss
* Make calculations involving repeated percentage change, not using the formula
* Use a variety of measures in ratio and proportion problems:
* currency conversion;
* rates of pay;
* best value;
 | **Compound Measures*** use kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question)
* change d/t in m/s to a formula in km/h, i.e. d/t × (60 × 60)/1000 – with support
* Find the original amount given the final amount after a percentage increase or decrease
* Use compound interest
* Set up, solve and interpret the answers in growth and decay problems
* Understand that X is inversely proportional to Y is equivalent to X is proportional to
* Interpret equations that describe direct and inverse proportion
 |
| **Percentages*** recognise proportions of a whole and use simple fractions and percentages to describe these
* understand percentage as ‘the number of parts in every 100’, and find simple equivalent fractions, decimals and percentages
* calculate simple fractions and percentages of numbers and quantities.
* Understand that a percentage is a fraction in hundredths
 | **Percentages*** use fractions and percentages to describe parts of shapes
* understand percentage as the ‘number of parts per 100’ and recognise equivalent percentages, fractions and decimals
* calculate simple fractions and percentage of quantities and measurements
* Find a percentage of a quantity without a calculator: 50%, 25% and multiples of 10% and 5%
* Find a percentage of a quantity or measurement (use measurements they should know from Key Stage 3 only)
 | **Percentages*** express one given number as a percentage of another
* calculate percentages and find the outcome of a given percentage increase or decrease
* Calculate amount of increase/decrease
* Convert between fractions, decimals and percentages
* Use percentages to solve problems, including comparisons of two quantities using percentages
* Percentages over 100%
* Find a percentage of a quantity, including using a multiplier
 | **Percentages*** Use percentages in real-life situations, including percentages greater than 100%
* Price after VAT (not price before VAT)
* Value of profit or loss
* Simple interest
* Income tax calculations
* Use decimals to find quantities
* Use a multiplier to increase or decrease by a percentage in any scenario where percentages are used
* Understand the multiplicative nature of percentages as operators
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| **RATIO AND PROPORTION 1** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Ratio, Proportion and Rates of Change*** Express the division of a quantity into a number parts as a ratio;
* Write ratios in form 1 : *m* or *m* : 1 and to describe a situation;
* Write ratios in their simplest form, including three-part ratios;
* Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
* Use a ratio to find one quantity when the other is known;
* Write a ratio as a fraction and as a linear function;
* Identify direct proportion from a table of values, by comparing ratios of values;
* Use a ratio to compare a scale model to real-life object;
* Use a ratio to convert between measures and currencies, e.g. £1.00 = €1.36;
* Scale up recipes;
* Convert between currencies.
* Solve proportion problems using the unitary method;
* Work out which product offers best value and consider rates of pay;
* Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
* Relate algebraic solutions to graphical representation of the equations;
* Recognise when values are in inverse proportion by reference to the graph form;
* Set up and use equations to solve word and other problems involving inverse proportion, and relate algebraic solutions to graphical representation of the equations.
* Recognise and interpret graphs showing direct and indirect proportion;
* Use and interpret maps and scale drawings, using a variety of scales and units;
* Read and construct scale drawings, drawing lines and shapes to scale;
* Estimate lengths using a scale diagram;
 | **Ratio, Proportion and Rates of Change*** Use *y* = *kx* to solve direct proportion problems, including questions where students find *k*, and then use *k* to find another value;
* Set up and use equations to solve word and other problems involving direct proportion;
* Identify direct proportion from a table of values, by comparing ratios of values, for *x* squared and *x* cubed relationships;
* Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of *k* in *y* = *kx*;
* Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when *A*:*B* are in the ratio 3:5, *A* is *B*. When 4*a* = 7*b*, then *a* =  or *a*:*b* is 7:4;
 | **Ratio, Proportion and Rates of Change** * Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
* Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
* Solve problems involving inverse proportionality;
* Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.
* Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
* Recognise, sketch and interpret graphs of exponential functions *y* = *kx* for positive values of *k* and integer values of *x*;
* Use calculators to explore exponential growth and decay;
 | **Ratio, Proportion and Rates of Change*** Set up, solve and interpret the answers in growth and decay problems;
 |
| **Compound Measures*** Understand and use compound measures and:
* convert between metric speed measures;
* convert between density measures;
* convert between pressure measures;
 | **Compound Measures*** Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question);
 | **Compound Measures** | **Compound Measures** |
| **Percentages*** Convert between fractions, decimals and percentages;
* Express a given number as a percentage of another number;
* Express one quantity as a percentage of another where the percentage is greater than 100%
* Find a percentage of a quantity;
* Find the new amount after a percentage increase or decrease;
* Work out a percentage increase or decrease, including: simple interest, income tax calculations, value of profit or loss, percentage profit or loss;
* Compare two quantities using percentages, including a range of calculations and contexts such as those involving time or money;
* Find a percentage of a quantity using a multiplier and use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
* Find the original amount given the final amount after a percentage increase or decrease (reverse percentages), including VAT;
* Use percentages in real-life situations, including percentages greater than 100%;
* Work out the multiplier for repeated proportional change as a single decimal number;
* Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation;
 | **Percentages*** Understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents, and choose fractions, decimals or percentages appropriately for calculations.
* Describe percentage increase/decrease with fractions, e.g. 150% increase means  times as big;
* Use calculators for reverse percentage calculations by doing an appropriate division;
 | **Percentages** | **Percentages** |

GCSE STATISTICS

*Threshold concepts for 5 yr Maths Course (Grades 1-9)*

*Version 29/05/17*

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| **STATISTICS 1** |
| **1** | **2** | **3** | **4** |
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* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Increasing confidence to reason, interpret and communicate mathematically** Grade 3 Students should, for some Grade 3 topics and most lower Grade maths, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Increasing independence when solving problems within mathematics and in other contexts** Grade 4 Students should, for most Grade 3 content and some Grade 4 maths, be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
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* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Presenting Data*** represent data in:

-tally charts-pictograms (symbol representing 1 or 2 units)-simple bar charts-simple Venn and Carroll diagrams.* represent data in Venn and Carroll diagrams, pictograms, bar charts and simple line graphs
* collect discrete data and record them using a frequency table, where appropriate using equal class intervals
* construct simple pie charts
* Plot coordinates in first quadrant and read graph scales in multiples
* Calculate total population from a bar chart or table
* Find greatest and least values from a bar chart or table
 | **Presenting Data*** construct frequency tables for gathering discrete data, grouped where appropriate in equal class intervals
* construct graphs and diagrams to represent data, including:

-bar-line graphs-frequency diagrams for grouped discrete data* construct, on paper and using ICT, graphs and diagrams to represent data, including:

-composite bar charts -dual/comparative bar charts for categorical and ungrouped discrete data-vertical line charts-line graphs-frequency diagrams for grouped discrete data-simple pie charts.* construct bar-line graphs and frequency diagrams for grouped discrete data, on paper and using ICT
* Interpret simple pie charts using simple fractions and percentages; ,  and multiples of 10% sections
 | **Presenting Data*** plan, construct and use two-way tables for recording discrete data
* construct:

-bar charts and frequency diagrams for grouped discrete data-pie charts for categorical data-simple scatter diagrams* construct stem-and-leaf diagrams plan, construct and use frequency tables with equal class intervals for gathering continuous data
* construct:

-bar charts and frequency diagrams for continuous data-simple line graphs for time series-stem and leaf (including back-to-back)-histograms with equal class intervals* Know which charts to use for different types of data sets
* Construct pie charts for categorical data and discrete/continuous numerical data
 | **Presenting Data*** construct frequency tables for gathering discrete or continuous data, choosing suitable class intervals
* calculate statistics and select those which address the questions posed, constructing stem-and-leaf diagrams where appropriate
* select, construct and modify, on paper and using ICT, suitable graphs and diagrams to progress an enquiry, e.g. frequency diagrams, pie charts
* collect and represent discrete and continuous data, using ICT where appropriate
* use statistical measures, tables and diagrams, for discrete and continuous data, using ICT where appropriate
* select, construct and modify, on paper and using ICT, suitable graphs and charts to progress an enquiry, including:

-line graphs for time series-scatter graphs to develop further understanding of correlation* Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts
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| **Averages*** use the mode and range to describe sets of data
 | **Averages*** find the mode, mean, median and range for a set of discrete data, and the modal class for grouped discrete data
 | **Averages*** construct and interpret stem-and-leaf diagrams, and compare two simple distributions using the range and one of the mode, median or mean
* Can interpret and find a range of averages as follows:
* median, mean and range from a (discrete) frequency table
* mode and range from a bar chart;
* median, mode and range from stem and leaf diagrams
 | **Averages*** Can interpret and find a range of averages as follows:
* range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table
* mean from a bar chart
* Understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values
* Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf
* Recognise the advantages and disadvantages between measures of average
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| **STATISTICS 1** |
| **5** | **6** | **7** | **8-9** |
| Students should be able to demonstrate fluency in lower grade mathematics and a developing ability to decision make and solve problems!***Fluency > Problem Solving******Higher Confidence*** | **Fluency - Extensive ability to use and apply standard techniques** Grade 6 Students should, for Grade 5 maths, be able to: * accurately recall facts, terminology and definitions
* use and interpret notation correctly
* accurately carry out routine procedures or set tasks requiring multi-step solutions
 | **Decision making- Extensive ability to reason, interpret and communicate mathematically** Grade 7 Students should, for most topics, be able to: * make deductions, inferences and draw conclusions from mathematical information
* construct chains of reasoning to achieve a given result
* interpret and communicate information accurately
* present arguments and proofs
* assess the validity of an argument and critically evaluate a given way of presenting information
 | **Problem Solving- Extensive ability to solve problems within mathematics and in other contexts** Grade 8 (for most) and 9 Students (for all GCSE maths) should be able to: * translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes
* make and use connections between different parts of mathematics
* interpret results in the context of the given problem
* evaluate methods used and results obtained
* evaluate solutions to identify how they may have been affected by assumptions made
 |
| **Presenting Data*** Know which charts to use for different types of data sets;
* Produce composite bar charts;
* Produce comparative and dual bar charts;
* Produce pie chart;
* Produce frequency polygons for grouped data;
* Produce frequency diagrams for grouped discrete data;
* Produce line graphs;
* Draw scatter graphs in terms of the relationship between two variables;
* Draw lines of best fit by eye;
* Identify outliers and ignore them on scatter graphs;
* Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem;
 | **Presenting Data*** Construct time–series graphs;
* Know the appropriate uses of cumulative frequency diagrams;
* Construct cumulative frequency tables, cumulative frequency graphs/diagrams;
* Produce box plots from raw data and when given quartiles, median and identify any outliers;
 | **Presenting Data*** Produce histograms with equal class intervals:
* Know the appropriate uses of histograms;
* Construct histograms from class intervals with unequal width;
* Use and understand frequency density;
* From histograms:

complete a grouped frequency table; | **Presenting Data** |
| **Averages*** Design and use two-way tables for discrete and grouped data;
* Use information provided to complete a two-way table;
* Sort, classify and tabulate data and discrete or continuous quantitative data;
* Calculate mean and range, find median and mode from a small data set;
* Use a spreadsheet to calculate mean and range, and find median and mode;
* Construct and interpret stem and leaf diagrams (including back-to-back diagrams):
* find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
* Calculate the mean, mode, median and range from a frequency table (discrete data);
 | **Averages*** Recognise the advantages and disadvantages between measures of average;
* Construct and interpret grouped frequency tables for continuous data:
* for grouped data, find the interval which contains the median and the modal class;
* estimate the mean with grouped data;
* understand that the expression ‘estimate’ will be used where appropriate, when finding the mean of grouped data using mid-interval values.
 | **Averages*** use a moving average to identify seasonality and trends in time series data, using them to make predictions
 | **Averages*** Apply the concepts of instantaneous and average rates of change by looking at the gradients of tangents and chords to a curve
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